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Retail pricing data combine multiple decisions (e.g., regular pricing and discounting) that are possibly made by multiple decision makers (e.g., retailers and manufacturers). For example, temporary price reductions (high-frequency price changes) can be used to price discriminate in the short run, whereas regular price adjustments (low-frequency price changes) might reflect changes in long-term costs or demand. Time disaggregation cannot disentangle these factors, because frequency aggregation exists even when data are analyzed at the lowest possible level of temporal aggregation. Because little is known about the nature of pricing interactions across various planning cycles, this article develops several empirical generalizations about the role of periodicity in pricing. Using week–store stockkeeping-unit-level price data in 35 grocery categories, the authors find that (1) cross-brand correlation in prices occurs at multiple planning horizons, and the planning horizon of the predominant interaction does not typically coincide with the sampling rate of the data; (2) aggregating pricing interactions across frequencies obscures distinct and different interactions; (3) pricing interactions are related to category- and brand-specific factors, such as mean interpurchase times; (4) regular price changes explain most of the variation in prices; and (5) periodicity can affect inferences about the nature of competition within a category. The authors conclude by discussing several practical marketing applications for which marketing decisions across frequencies have relevance.

The Periodicity of Pricing

Empirical research pertaining to the measurement and prediction of competitors' price interactions is pervasive in the marketing literature (e.g., Gatignon 1984; Hanssens 1980; Lambin, Naert, and Bultez 1975). More recently, considerable attention has been devoted to the dynamics inherent in competitor price interactions (Dekimpe and Hanssens 1999; Leeflang and Wittink 1992, 1996, 2001; Nijs et al. 2001; Steenkamp et al. 2005). This work has led to a richer understanding of competition, engendering the view that response may occur with some delay. Our aim is to contribute to this literature by considering how pricing interactions differ across various periodicities. Lately, this issue has received increased interest in the marketing literature. Kadiyali, Sudhir, and Rao (2001, p. 177) note that "periodicity of decision making and time aggregation/disaggrega-

tion are important issues to bear in mind," and Pauwels and colleagues (2004, p. 176) add, "as there is increasing evidence that the same relationship need not hold among two variables at different frequencies, various substantive marketing problems may warrant further investigation along that dimension."

Weekly pricing data embed both regular price decisions (which may change on an infrequent basis) and temporary price reductions (which may occur more frequently). Although these multiple decisions are agglomerated into a single pricing series, the goals of short- and long-term pricing, as well as consumer response to them, may be different (Shaffer and Zhang 2002). For example, a manufacturer might use discounts to collude against weaker brands (Lal 1990), price discriminate among brand switchers (Farris and Quelch 1987), or increase consumption. In contrast, regular price changes might reflect changes in overall cost structure, demand, or market structure. Likewise, retailers might choose to use discounts to drive category profitability or store choice or to shift inventory costs to consumers (Blattberg, Eppen, and Lieberman 1981). Contributing to differences in the periodicity of pricing are the response latencies inherent in pricing interactions. Although retailers can change prices fairly readily, manufacturers' ability to

implement price changes at retail can take considerably longer. According to an industry expert we interviewed, manufacturer price changes can take six months to filter into retail prices. In one example, this expert noted that changes in on-pack pricing were necessary to change prices at retail, leading to price implementation delays arising from the production and distribution time associated with relabeling. Previous research has further documented that manufacturer price changes can adjust more slowly than retail price, typically taking 5–13 weeks (Kopalle, Mela, and Marsh 1999; Leeflang and Wittink 1992, 2001). An exception to this generalization exists when manufacturers can obtain competitive dealing schedules from their retailers well in advance of the deals (our communications with various manufacturers made this apparent). This enables firms to have sufficient time to anticipate and react to price changes at retail.

In light of the preceding discussion, we explore differences in pricing interactions across frequencies using a large grocer database spanning 35 categories and 166 brands to develop answers to questions such as the following:

- Is price variation more common at some frequencies than at others? For example, do regular price changes explain more of the variation in prices than discounts?
- Are pricing interactions between brands common across all periodicities, or are there dominant frequencies for pricing interactions? Does the nature of these interactions differ across frequencies? Knowledge of the dominant pricing frequencies is helpful to determine the appropriate frequencies for modeling (competitive response to) prices and assessing the potential for frequency aggregation biases.

- Are the frequencies at which prices vary and/or interact related to differences in brand and/or category factors, or are these frequencies common across brands and categories? The frequencies at which prices vary and/or interact lend insight into decision-making frequencies and goals of various pricing agents.
- How do price variation and price covariation contrast across frequencies? Do the predominant pricing frequencies and interaction frequencies align? To the extent that these differ, brand-specific pricing factors can differ in relative importance from the prices of competing brands.
- What are some of the potential implications of frequency aggregation bias for inferences about competitive response?

To exemplify some of these issues, Figure 1 depicts more than four years of retail prices per can for two brands of beer (Budweiser and Miller) in a Dominick's Finer Food (DFF) store in Chicago. As is clear from the graph, there are multiple frequencies represented in the price data. First, there are higher-frequency oscillations in pricing (occurring every 4 weeks or so), which are negatively correlated for Budweiser and Miller ($r = -.44$).[1] To the extent that short-term price variation is retailer driven, this would suggest that the retailer tends to promote in alternate weeks (Krishna 1994). Second, there are lower-frequency price fluctuations between Budweiser and Miller (occurring every 25 weeks or so) associated with long-term movements in regular price, which are strongly positively correlated ($r = .97$). However, despite these strong correlations in

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[1]For this illustration, we isolated the short- (long-) term variation in the data using a high- (low-) pass filter that eliminates variation below or above quarterly frequencies.

Figure 1

SKU PRICES FOR BUDWEISER AND MILLER BEER 12-OUNCE CANS ACROSS 225 WEEKS
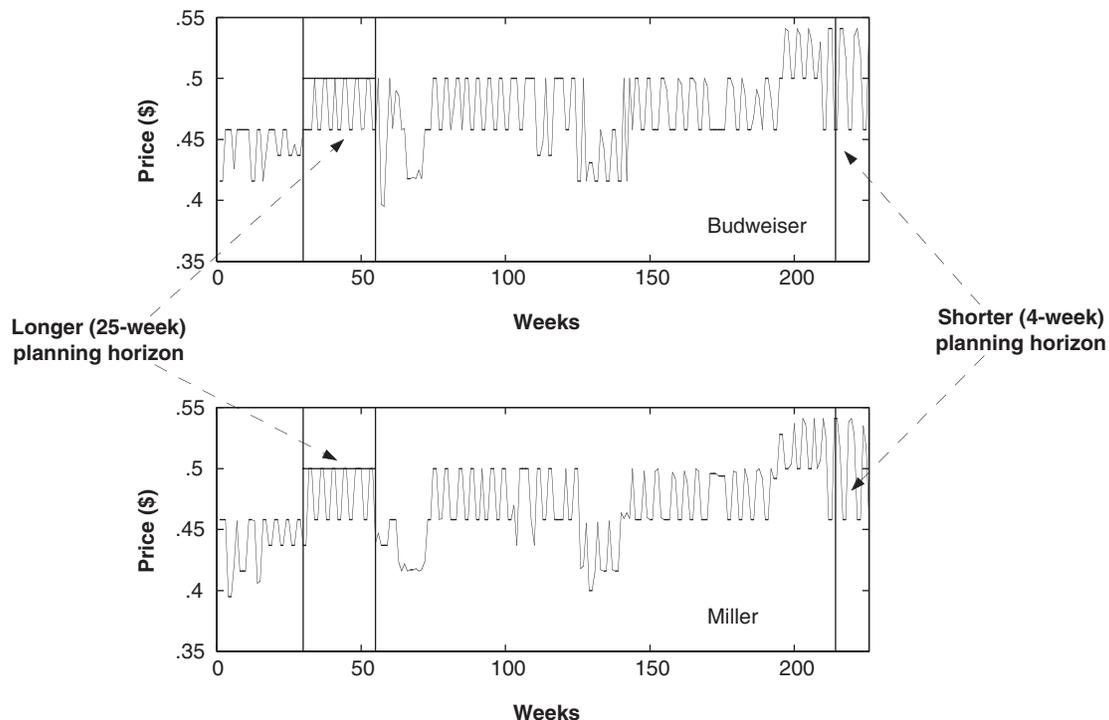
the short- and long-term pricing, the overall comovement of these two price series is relatively weak (r = .24). This simple example provides some descriptive evidence that (1) price variation exists at multiple frequencies, (2) the nature of these interactions differs across frequencies, and (3) aggregating across them can obscure these interactions. We find similar patterns in other categories.

As do Pauwels and colleagues (2004), we contend that little is known about pricing in the frequency domain relative to the vast literature on price interactions in the time domain. In addressing these issues, we note that periodicity aggregation is distinct from time aggregation (Leone 1995). Even when data are disaggregated to the shortest data interval, multiple decision makers and multiple decisions remain combined in the data. Indeed, as data are sampled at higher frequencies (e.g., weekly versus monthly), more interactions become intermingled in the pricing series. As such, the literature on time aggregation provides limited insights into the periodicity of decision making.

This article proceeds as follows: Next, we outline the methods used to illustrate our points. Then, we discuss the data and present the results of the analysis. A key finding is that, empirically, the periodicity of price interactions differs in general from the sampling rate of the data. We explore further implications of our findings by showing that the frequency of price interactions can affect statistical inferences about the nature of price competition. Finally, we offer our conclusions.

METHOD

To assess the influence of periodicity on competition, we employ a spectral decomposition of price series. This descriptive analysis is intended to show how the magnitude and direction of price interactions depend on a given range of frequencies, which we denote as "planning horizons." In a second-stage analysis, we use regression methods to determine how the magnitude and direction of price interactions depend on various brand- and category-level variables.

Spectral Decomposition of Price Covariation

Spectral analysis is a technique that creates a (co)variance decomposition of price data into price cycles at different frequencies.[2] This technique has been widely applied in economics and finance (e.g., Andersen and Bollerslev 1997). Although it is used only rarely in marketing (Chatfield 1974; Parsons and Henry 1972), its potential to yield insights into the periodicity of decision making has led to increased calls for its application to marketing theory (Deleersnyder et al. 2004; Lemmens, Croux, and Dekimpe 2004; Pauwels et al. 2004). The technical aspects of spectral decomposition are well documented (e.g., Hamilton 1994). Therefore, we relegate all technical aspects of our exact implementation to Appendix A.

We first consider the "population or power spectrum" of the pricing series. The spectrum of a pricing series reveals its proportion of variance at different frequencies. The total variance of prices can be viewed as the sum of the variance

components across the different frequencies. When pricing is standardized, the resulting variance components sum to one across frequencies, and the interpretation of the power spectrum becomes one of a "spectral distribution," in which the value of the spectrum approximates the fraction of variance that occurs at a given planning cycle. For purposes of notation, let $s_i^c(\omega)$ denote the spectral power for the price of brand i in category c and frequency ω. Given the time-domain data in Figure 1, we expect to observe a strong spectral component for the beer data at $\omega \approx 4$ weeks and at $\omega \approx 25$ weeks. Panels A and B of Figure 2 confirm this expectation. Figure 2 depicts the standardized spectra for the Budweiser and Miller prices. The horizontal axis depicts the pricing cycle, or planning horizon (in weeks), which ranges from a high of 50 weeks to a low of 2 weeks.[3] The vertical axis in Panels A and B is the power spectrum standardized in sum to one. The box lots in the graphs represent the 5%, 25%, 50%, 75%, and 95% percentiles in the sample distribution of the spectral estimates in the beer category at each planning horizon (Appendix A outlines the procedure for estimating these percentiles). The depicted power spectra reveal considerable pricing variance at $\omega = 4$ and $\omega = 25$.

Just as the pricing variance can be apportioned to different frequencies using the decomposition of the spectrum, covariance can be apportioned using the "cross-spectrum." This yields a measure called "coherence," which captures the squared correlation between two pricing series at a particular frequency (see Hassler 1993). The coherence values range from 0 (no interaction between two competitors at a particular frequency) to 1 (very strong interaction at a particular frequency). For purposes of notation, let $h_{ii'}^c(\omega)$ denote coherence for the prices of brand pair {i, i′} in category c and frequency (or periodicity) ω. Similar to a correlation measure, the coherence measure is symmetric, that is, $h_{ii'}^c(\omega) = h_{i'i}^c(\omega)$. Importantly, coherence measures the presence of correlation in prices at a particular frequency, whether competitor price interactions are instantaneous or lagged. This attractive property ensures that there is no confound between price reactions and the timing of those reactions.

Continuing with the beer example, Panel C in Figure 2 portrays the coherence between Budweiser and Miller using the data in Figure 1 and controlling for the other major brands' prices (i.e., the pricing analysis is multivariate), including Coors. Figure 2 indicates two areas of high coherence between Budweiser and Miller at 4 and 25 week cycles. Thus, the combination of the spectra and coherences reveals that the interaction of prices between Budweiser and Miller occurs at two empirically important price cycles. However, as we indicated in our discussion of Figure 1, prices comove negatively at high frequencies (r = −.44) and positively at low frequencies (r = .97), so the comovement across the aggregation of these price cycles (which is the time series of the prices) is small (r = .24).
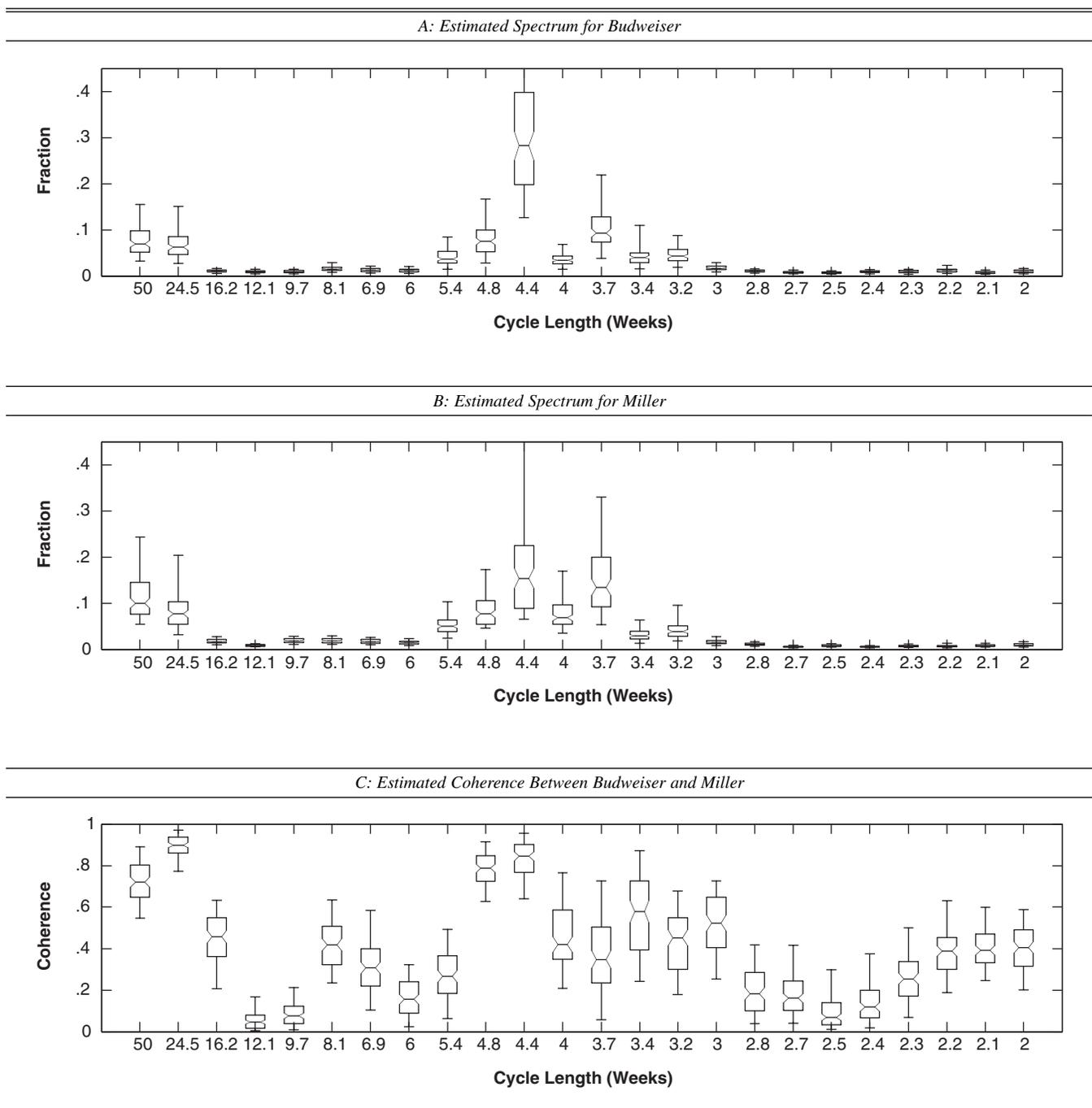
In summary, the finding of a strong comovement at specific frequencies but little comovement in aggregate illustrates the potential pitfalls of frequency aggregation. From

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[2]Cycles (or periods) and frequencies are inversely related. That is, letting T denote the length of the price cycle (counted in time units) and ω denote frequency (expressed in radians), we can express the relationship between planning period and frequency as $T = (2\pi/\omega)$, $0 < \omega \leq \pi$ (see also Harvey 1975).

[3]The low end of the range is set at two weeks, which implies 26 price cycles per year. Because prices are set weekly, the shortest complete price cycle for weekly data is biweekly and, thus, the highest frequency observable in the data.

Figure 2

SPECTRAL DECOMPOSITION AND COHERENCE OF SKU PRICES FOR BUDWEISER AND MILLER BEER 12-OUNCE CANS

A: Estimated Spectrum for Budweiser

B: Estimated Spectrum for Miller

C: Estimated Coherence Between Budweiser and Miller

an aggregate correlation, it might be concluded that the prices of Budweiser and Miller are independent or only weakly related when, indeed, there are substantial patterns of price covariation observed across different frequencies. An alternative approach, which we take here, is to consider each frequency differently when assessing the degree of pricing interactions.

Regression

After the spectral decomposition of the data, the next step of our analysis assesses whether pricing variation, as measured by the spectrum, and competitor interactions, as meas-

ured by coherence, systematically vary across planning horizons, category characteristics, and brand characteristics. Next, we explain how we do this.

Step 1: Categorize the continuous periodicities (ω) into a discrete number of planning horizons (p = [long, medium, short]). Following the work of Leeflang and Wittink (1992), we define short-term reactions as those that occur at intervals of 4 weeks or less (monthly). Leeflang and Wittink (1992, 2001) note that this cutoff corresponds roughly with the period in which manufacturers cannot adequately respond to observed changes in retail price activity. We denote medium-term reactions as those that occur between

the 4-week period and the 13-week (quarterly) period. Such price movements are more likely to include manufacturer reactions to competitors' discounting policies and may include changes in regular price. We define price changes that occur at greater than a quarterly frequency as longer term, and these may be more reflective of long-term strategic objectives. Finally, we disregard price changes that occur with a periodicity of more than 50 weeks to ensure a sufficient number of cycles to produce a reliable analysis (with eight years of data, this represents roughly four cycles). In addition, excluding annual cycles in prices (52 weeks) reduces the risk of inadvertently confusing positive covariation in prices due to common seasonality in costs with positive covariation arising from strategic long-term price matching. Analyzing time series in the frequency domain lends itself naturally to the exploration of seasonality effects, which manifest as spikes in the spectrum at the frequency of the seasonality. For example, in the soup category, a cyclical demand that correlates with the winter season might be expected.

Step 2: Compute the coherence and spectrum for these discrete planning horizons. Within each planning horizon (short, medium, long), the coherences and power spectra exist for all frequencies. To summarize coherence into a single value for each planning horizon, we compute a weighted average of the coherence, in which the weights are the inverse of the sampling error in the estimates of these statistics at each frequency (for details, see Appendix A).[4] To summarize the spectra into a single value for each planning horizon, the natural measure is the total fraction of price variation that falls in each planning horizon.

Step 3: Regress planning horizon, brand, and category characteristics on coherence. Using regression analysis, we investigate whether there are important differences in coherence across brand pairs, categories, and planning horizons. Knowledge of these differences is useful in predicting when periodicity matters in terms of making inferences about pricing interactions. Appendix B provides details about the specification and estimation of the regression model. Note that the regression accounts for the unobserved heterogeneity in brand pairs and in categories.

DATA

We used the DFF database for this research (http://gsbwww.uchicago.edu/kilts/research/db/dominicks/). The DFF data comprised 400 weeks of store movement data and thus are well suited to the study of long-term variation in retail prices. The DFF data contain 29 categories of consumer packaged goods (CPG), though many of these categories contain multiple subcategories (e.g., grooming products contain razors, shaving cream, and deodorant, among other subcategories). In total, we conducted spectral analyses on 35 subcategories.[5]

Next, we defined a price index for each brand using the price the brand charged for the "leading item" in the category. From the many Universal Product Codes (UPCs) within a category, we defined the "leading item" as having both high levels of demand within the category and a long duration in the data, so observations for prices are rarely missing for these items. For example, in bottled juices, we selected a UPC related to a particular size (64 ounce) and a particular type of juice (apple) for each of the major brands. The prices of similar stockkeeping units (SKUs) within a brand over time are highly correlated. For example, the case price of Budweiser and Budweiser Light are correlated at .99 over the data. As such, to the extent that complete pricing is available for different SKUs, the pricing analysis is typically invariant to the SKU selected as the leading SKU.

We used item-level prices to form the brands' price indexes, as opposed to aggregating the brands' prices across UPCs, to avoid introducing a spurious high-frequency component from (1) aggregating across nonsynchronized price changes and (2) weekly variation in aggregation weights (e.g., sales). Selecting the same "leading items" across brands within a category further ensures that we are most likely to observe price interactions where they exist.

We selected prices from one store (Store 44) because the number of missing weeks of pricing was small for this store. Although we used pricing data from one store, DFF's use of pricing zones implies that the pricing behavior reflects its pricing in many stores. Store 44 is in Zone 2, which comprises 29 of the 88 stores in the data. We use Store 128 for beer because there are no observations for beer in Store 44. A limiting factor is that our use of one store precludes us from explicitly analyzing interstore pricing behavior.

There are a few missing observations scattered about in the UPC pricing data. When these occur, we set the missing prices equal to those of the nearest period. This interpolation approach ensures that prices more closely match the modal prices for regular and sale prices rather than some point in between.[6] In exceptional instances, the series for one UPC is too short. This can occur when a SKU is discontinued and replaced by a similar SKU or when the retailer stops shelving the SKU. These gaps in prices can last for months or quarters. In such cases, we resorted to an average of the prices for the most similar within-brand UPCs. These series are selected to correlate close to 1.0 for periods in which they overlap, so this construction should not attenuate pricing variability.

The number of price series per category ranged from 3 to 9. The 35 categories yielded 166 price series and 348 pairs of price series. Therefore, for the second-stage analysis, we have 166×3 planning horizons, or 498 observations of the power spectra, and 348 pairs $\times 3$ planning horizons, or 1044 observations of coherence.

Although we focus on retail prices in our analyses, it is also possible to use wholesale pricing data to analyze pric-

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[4]We also computed an alternative measure by selecting the most important coherence for each planning horizon. We define this measure as the coherence at the frequency that maximizes the sample mean of coherence over the sample standard deviation of coherence. This measure yields identical insights.

[5]Analgesics, bath soaps, candy bars, cereal, gum, soup, conditioners, cookies, cola, deodorant, fabric softener sheets, fabric softener liquids, frozen dinners, frozen entrées, frozen orange juice, graham crackers, apple juice, liquid dish detergent, liquid laundry detergent, liquid soaps, oatmeal, refrigerated orange juice, paper towels, toilet paper, razors, beer, saltines,

shredded cheese, shaving cream, shampoo, sliced cheese, snack crackers, toothbrushes, toothpaste, and tuna fish.

[6]We also tried a c-spline interpolation, and the results were virtually identical. Because the number of missing observations is small, alternative common methods of interpolation probably have inconsequential effects on our results.

ing behavior. We refrain from doing so for three reasons. First, many firms have access only to retail prices, so the most useful approach for these firms focuses on retail prices. Second, by excluding retailer behavior, the wholesale data omit an important player of interest to many firms. Third, the DFF wholesale prices are not manufacturer prices to the retailer but rather reflect a weighted average cost of inventory. As such, it is not purged of retailer behavior, because it includes the effect of retail sales data and accounting procedures, inhibiting the likelihood of isolating manufacturer pricing behavior.

RESULTS

Components of Price Cycles and Interaction Among Prices

To assess whether the spectral distributions predominantly vary with planning horizon (common to brands and categories) or brand/category factors (common to frequencies), we first discuss a variance decomposition of the spectra and coherence measures along planning horizons, categories, and brands. This analysis gives a description of the most salient factors that explain the prevalence of price cycles.

Components of price cycles. We conduct the following decomposition of the spectral values $s_i^c(p)$, for brand i, category c, and planning horizon p:

$$(1) \qquad s_i^c(p) = a_p + b_{cp} + e_{icp}.$$

Note that there are no brand and category fixed effects. The spectra sum to 1 across frequencies for each brand/category, and thus brand and category main effects cannot explain variation in $s_i^c(p)$. Using our previous definition of short, medium, and long cycles, we find that the fixed effects of planning horizon, a_p, p = 1, 2, 3, explain 50% of the variation in the price spectra. This result implies that brands share similar patterns in their price spectra. In particular, most price variation occurs in the medium term (4–13 weeks), and the least price variation occurs in the short term (2–4 weeks). This emphasis on slower pricing cycles is surprising in competitive industries, such as detergents, frozen dinners, and cereals, in which promotions are endemic. This finding is especially relevant to researchers who deemphasize the role of regular price variation in modeling sales response (e.g., Kopalle, Mela, and Marsh 1999) because it suggests that this practice omits much pricing variation from the analysis. Next, adding the "category–frequency" interactions, b_{cp}, to the model increases the R-square to 77%. This category-specific effect suggests the existence of a category-specific cadence in pricing decisions common to all the brands within a category. The remainder of the variation in $s_i^c(p)$, 23%, is specific to brand/frequency. In summary, we observe that planning horizon and category pricing variation (77%) is more prevalent than brand-specific pricing variation (23%).

Our inspection of the complete spectra $s_i^c(\omega)$, $\omega = 0, ..., \pi$, across brands and categories yields an additional empirical generalization: The mass in the spectra is typically concentrated in relatively few planning cycles; specifically, the top 20% of frequencies in terms of spectral density mass account for more than 50% of the variation in prices. As such, the bulk of price variation arises from relatively few dicrete price cycles that are category specific. This last point is well illustrated in Figure 3, which lists category

averages spectra for four exemplar categories.[7] These graphs indicate that the majority of variation in the price data is concentrated in few planning horizons, especially in the beer and the analgesics categories. Shredded cheese exhibits two smaller modes of price variation, one at less than 4 weeks and another at greater than 13 weeks. Importantly, this suggests that frequencies of interest can be isolated for detailed analyses of competitor interactions.

Price interactions at different price cycles. Analogous to the fixed effects model in Equation 1, the principal dimensions along which coherence, $h_{ii'}^c(p)$, varies across brand pairs ii´, categories c, and planning horizons p can be described by the following variance decomposition:

$$(2) \qquad h_{ii'}^c(p) = a_p + b_{cp} + d_c + e_{ii'cp}.$$

Unlike the spectral density regression in Equation 1, the sum of $h_{ii'}^c(\omega)$ across ω in Equation 2 differs across categories. Therefore, we add a category fixed effect.

We find that the effects of planning horizon explain only modest proportions of the variation in the price interactions. Approximately 15% of variation in the cross-price interactions stems from the a_p (i.e., the common frequency effects across categories). Thus, there is no common pattern of coherence for the planning horizons across the categories or brand pairs. In contrast, the category main effects, d_c, explain 37% of the variation in coherence, suggesting that differences in coherence can be explained by the nature of the categories in which brands compete. The two dimensions (frequency and category) are orthogonal, so their combined effects account for 52% of the variation in coherence. Considering the interactions between the frequencies and the categories increases the variance explained to 57%. These results indicate that frequencies of price interactions are more brand-pair specific than frequencies of price variation, which tend to be common across brands. However, as with the spectra, the coherence evidences large components of variance in the data at the category level. Figure 4 shows several examples.

The analgesics category has especially high coherence in the lower-frequency part of the spectrum, indicating that competitor prices strongly interact at a quarterly and semiannual level in this category. The fabric softener category has low coherence throughout, indicating less overall price interaction. The beer category, as we discussed previously, has a strong interaction in the monthly price cycles but also in the semiannual price cycles. Finally, the shredded cheese category has a strong interaction at 4-week price cycles (during which the spectral distribution was also high), but the same pattern of coherence is not as distinctly reflected in the 14-week cycle (during which there was another mode in the spectra). This pattern indicates that pricing interactions might be more independent in the long term for shredded cheese, a conjecture we discuss in greater and more formal detail subsequently.

Collectively, the elements of Figure 4 suggest that empirical interactions in price occur at multiple price cycles and that they occur at all periodicities. These patterns

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[7]Note the different scales of Figures 2 and 3. Each box in Figure 2 sums across multiple cycle lengths in Figure 3. We refrained from plotting whisker boxes for all cycle lengths to avoid cluttering the graph.

Figure 3
EXAMPLES OF SPECTRA IN FOUR CATEGORIES
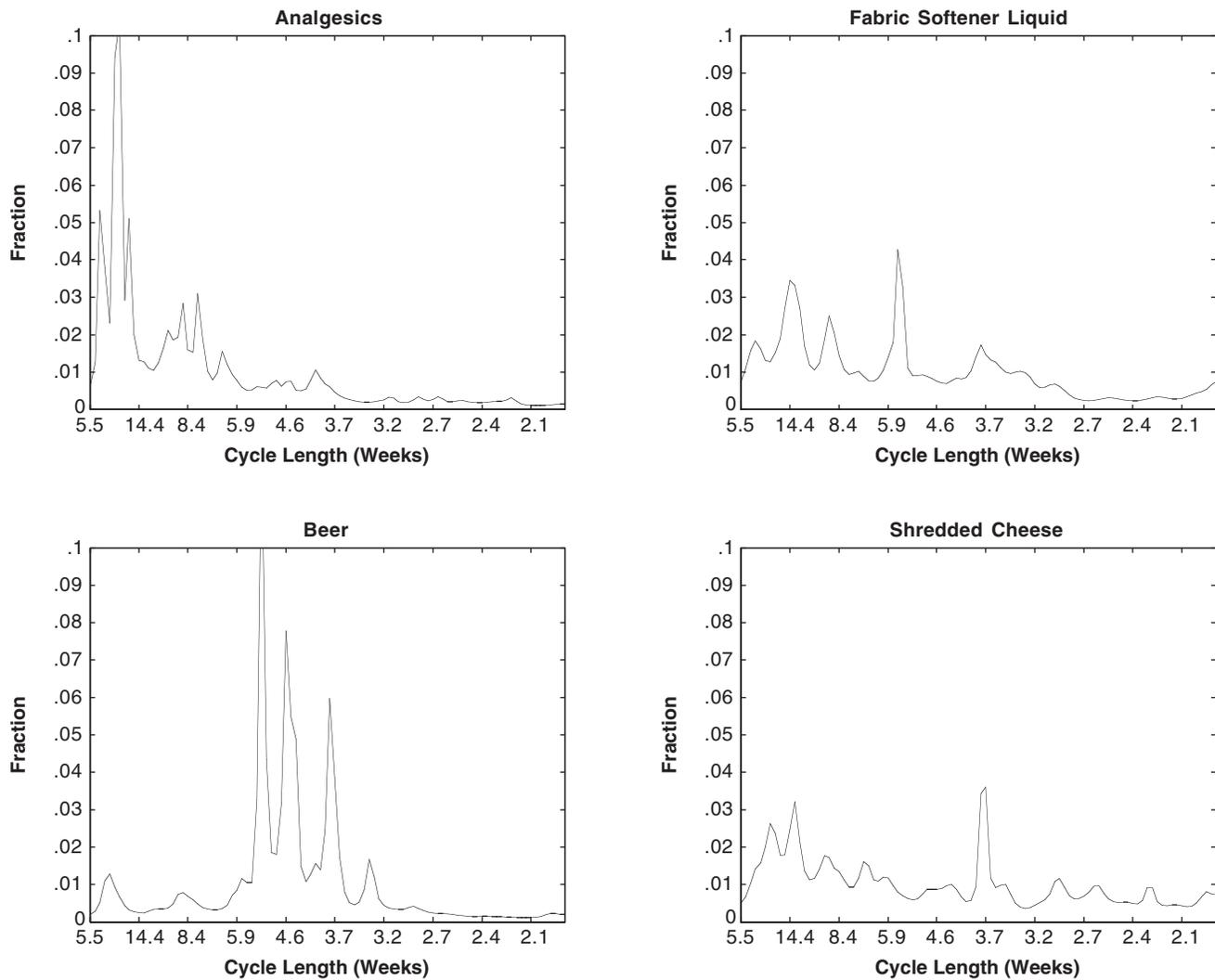
reflect patterns in the other categories as well. Thus, a key point from these descriptions of coherence is that in almost all 35 categories studied, the important frequencies of price interaction do not coincide with the sample rate of the data (weekly). This finding is especially important given that, to our knowledge, all analyses of competition focus on variation in price at the sampling rate of the data (which is often weekly).

How Do Empirical Interactions in Price Depend on the Brand, Category, and Planning Horizon?

Whereas the previous section found significant brand-pair and category components to coherence, in this section, we ask whether there are specific brand-pair and category characteristics that are associated with high or low coherence. To this end, we first explain the descriptor variables used and their hypothesized effects on coherence. Then, we discuss the empirical results for coherence. Finally, we replicate this analysis for the spectra.

Brand-level influences on competitor price interactions. The list of descriptors we consider here is not exhaustive; rather, it reflects the confluence of the measures available in the data and prior findings in the pricing literature. Because the list of descriptors is not exhaustive, we also account for unobserved factors at the category and brand level (see Appendix B). We hypothesize that several factors at the level of brand pairs affect the pricing interactions. We denote these brand-pair-level variables (which we define in Appendix C) as $\mathbf{z}_{ii'}^c$, and they are as follows:

- *Within-firm effects.* Reflective of common category planning practices, we expect multiproduct manufacturers to coordinate prices for their SKUs across different brands within a category. This would lead to higher coherence in prices among SKUs of different brands in the category produced by the same manufacturer.
- *Between-firm effects.* Price and quality tiers can also affect price interactions (Blattberg and Wisniewski 1989; Bronnenberg and Wathieu 1996). In general, Leeflang and Wittink's

Figure 4
EXAMPLES OF COHERENCE IN FOUR CATEGORIES
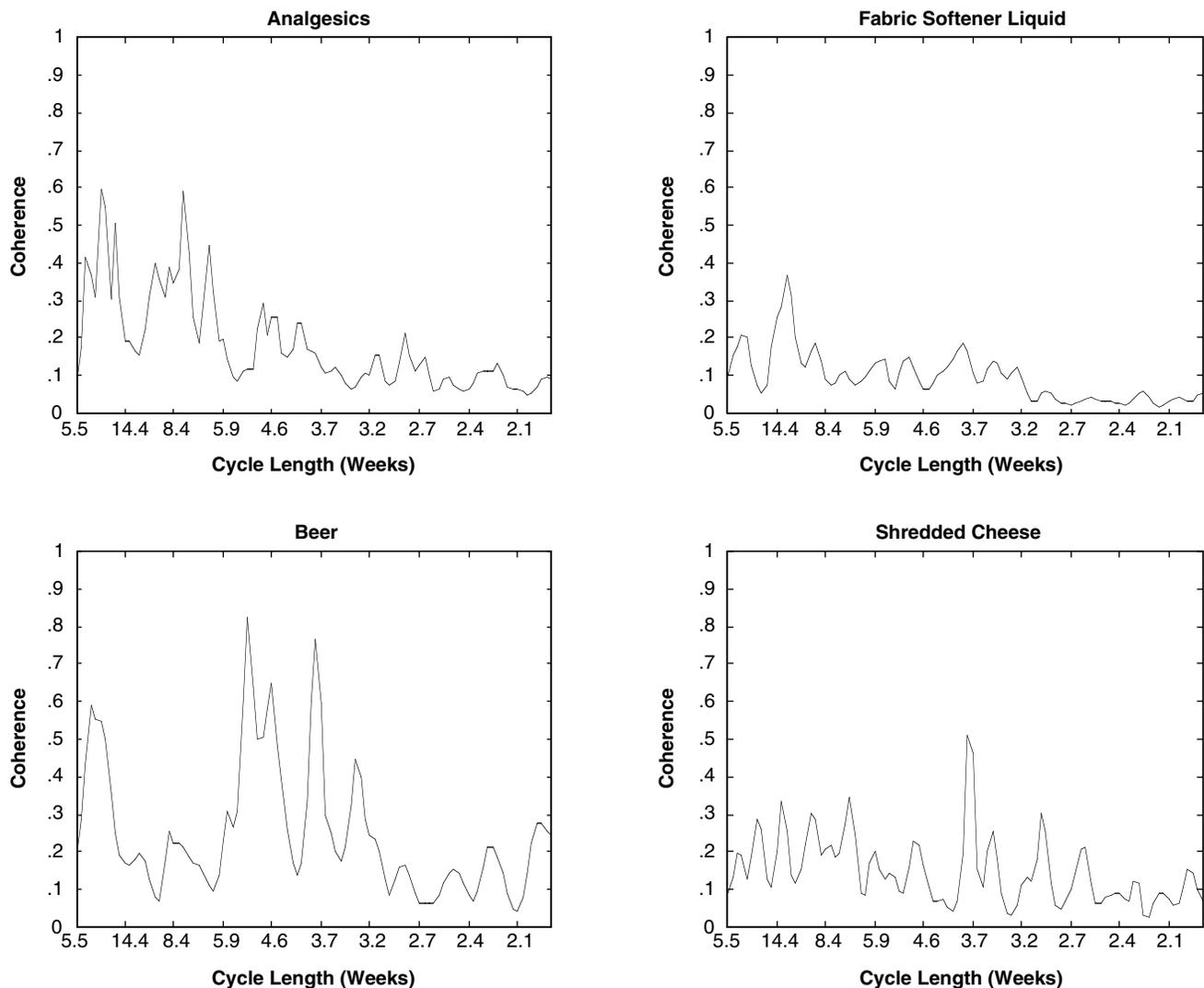
(2001) survey of managers indicates that competitor interactions are more likely when brands are positioned similarly.

➤*Private label*. Some private-label store brands are in different quality tiers than national brands (Hoch 1996). For these brands, prices are likely to be independent relative to more similar brand pairs (thus, the coherence should be lower). Other store brands are positioned more closely to the leading national brands (Dhar and Hoch 1997; Sayman, Hoch, and Raju 2002), suggesting greater coherence in price. A priori, it is not clear which effect will dominate, so we make no predictions about the effect of private label on coherence.

➤*Price differential*. We expect coherence to be limited for disparately priced brands, given their differential positioning.

➤*Firm size/revenue differential*. When large firms are confronted by pricing action from smaller firms, they may not perceive the attack as credible or important and therefore

may be more likely to ignore the small firms' pricing (Clark and Montgomery 1998). Accordingly, increased revenue differences may also lead to more independence in pricing and, thus, lesser coherence. Furthermore, to the extent that large and small brands are differentially positioned (e.g., small brands occupy a niche), the coherence is likely to be lower.

Category-level influences on competitor price interactions. In addition to brand descriptors, we delineate a set of category descriptors, denoted as $\mathbf{v}_c$, that we expect will moderate price interactions. We draw on the ability–motivation framework that Boulding and Staelin (1995) propose, which extends Heider's (1958) individual-level model to the firm level, to make predictions about the relative independence or coordination of pricing across categories. In the context of pricing, this theory suggests that

price interactions are more likely, to the extent that it is easy to interact (ability) and the effect of interactions on firm outcomes is high (motivation). We relate ability and motivation to the following variables (which we define in Appendix C); all else being equal, we should observe stronger effects with increasing ability or motivation.

- •*Industry characteristics.* We consider the following industry factors:
  - ➤*Concentration.* A dearth of competing products facilitates firms' ability to monitor competitors and enables them to set their prices more reliably (Gale and Branch 1982; Kuester, Homburg, and Robertson 1999). Thus, we hypothesize a positive relationship between concentration and coherence.
  - ➤*Volatility.* Highly volatile markets with a larger percentage of products exiting and entering the market can decrease the ability of organizations to monitor price activity, and this may reduce coherence in pricing.
- •*Retailer characteristics.* Retailer characteristics may also have consequences for competitor interactions.
  - ➤*Private-label share.* As private-label share increases, the share and revenue of national brands decrease. Therefore, there is less motivation for manufacturers to coordinate prices. Accordingly, coherence should decrease for the manufacturers that constitute the bulk of the market. In contrast, the importance of the category to the retailer should increase. As such, we might expect that the decrease in coherence is not as large for discounts (high-frequency price variation) as it is for regular price changes, because retailers tend to have a greater role in setting discounts than regular price changes.

- •*Demand characteristics.* Increased consumer price sensitivity is indicative of lower margins and, thus, profits. Accordingly, when price elasticities are higher, the motivation to coordinate prices is lower. This leads to increased independence of prices and, thus, lower coherence.
  - ➤*Penetration.* Increased penetration has been associated with greater price sensitivity (Narasimhan, Neslin, and Sen 1996). If this association holds, we expect penetration to contribute to a decrease in coherence.
  - ➤*Interpurchase time.* Bell, Chiang, and Padmanabhan (1999) find that increased interpurchase time leads to increased price sensitivity. Thus, we expect longer interpurchase times to decrease coherence.[8]

Regression results: coherence. Table 1 presents the results of the coherence regression, along with the parameter signs implied by the foregoing propositions. Table 1 indicates that empirical interactions in price depend significantly on the brand, category, and planning horizon. Overall, the fit of the model with the data is good ($R^2 = .41$), and the significant results are consistent with our expectations.

Several results merit discussion. With respect to the brand-level variables, we find that pairs of SKUs from the same manufacturer have high coherence in pricing. As we

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[8]Other factors may influence price sensitivity as well (Bell, Chiang, and Padmanabhan 1999). However, our attempts to measure them with a survey approach yielded variables that are redundant with the information contained in the aforementioned variables. For example, stockpilability was highly correlated with interpurchase time.

Table 1

REGRESSION RESULTS EXPLAINING THE COHERENCE FOR BRAND PAIRS IN THE SHORT, MEDIUM, AND LONG TERM

Variable (Expected Sign)	Parameter Values by Planning Horizon								
	Long	t		Medium	t		Short	t	
Intercept	–1.21	–18.11	**	–1.58	–23.78	**	–2.17	–32.60	**
Brand-Level Variables									
Same manufacturer (+)	.95	8.28	**	.94	8.15	**	1.09	9.43	**
Private label (?)	–.01	–.05		–.05	–.41		–.09	–.77	
Price differential (–)	–.07	–1.48		–.10	–1.98	*	–.06	–1.21	
Revenue differential (–)	.04	.78		.02	.50		–.02	–.32	
Category-Level Variables									
Industry Characteristics									
Concentration (+)	–1.21	–.52		–1.18	–.51		2.66	1.15	
Volatility (–)	–.01	–.95		.00	–.33		.01	.55	
Retailer Characteristics									
Private label share (–)	–3.19	–3.03	**	–2.62	–2.49	*	–2.40	–2.28	*
Demand Characteristics									
Penetration (–)	–.01	–.95		–.16	–.33		.03	.06	
Interpurchase time (–)	.18	.37		–.41	–.75		–1.24	–2.25	*
Random Effect Variances									
Brand-pair effects, σ_ε^2	.58	21.23	**						
Category/periodicity effects, σ_η^2	.28	7.52	**						
Model error, σ_ξ^2	.38	35.70	**						
Model Fit									
R^2	.41								
Log-likelihood	–900								

*$p < .05$ (two-tailed).
**$p < .01$ (two-tailed).
Notes: Negative effects denote lower coherence.

expected, multibrand firms coordinate price changes of different brands within the same category. This effect on coherence is positive in all three time horizons.

The effect of price differential is negative, as we expected, but only significantly so for the medium term. Thus, brands that are in different price tiers tend to have less coordination in prices, especially in regular prices.

With respect to category-level characteristics, the effect of private-label share is strongly negative, but less so in the short term. This result is consistent with the ability–motivation framework we outlined previously (i.e., a reduced motivation to coordinate prices).

Finally, as we expected, for products with high interpurchase times, there is less coherence, but only for short-term pricing. That is, frequent promotional price changes are more likely to be coordinated across brands in categories that have a low interpurchase time. This result is also consistent with the ability–motivation framework insofar as purchase frequency increases category size and, putatively, motivation.

Regression results: spectrum. We can specify a similar regression for the spectra, with some modifications. First, because the spectra sum to one, we use an attraction model to analyze the portion of variance in a particular planning horizon. Second, because of the same sum constraint, there is no main effect of category or brand descriptors. Third, because most of the variation in the spectra is explained by planning-horizon main effects, we focus on only a few other informative variables. Table 2 presents the results of the regression.

Two results are particularly noteworthy. First, the effect of brand size, standardized within category, is larger in the short term than in the long or medium term. This means that larger brands in a category have more variance in short price cycles than smaller brands. In other words, larger brands display larger short-term price changes than the smaller brands, possibly serving as retail loss leaders. Second, compared with categories with short and medium interpurchase times, long-term price cycles are more prevalent for categories with a long interpurchase time. Thus, as a simple heuristic, the importance of price cycles of different planning horizons seems to follow consumers' interpur-

chase times. Finally, note that the first effect is a within-category (i.e., brand) effect, whereas the second is a cross-category effect. As such, the two are independent. In summary, using a large database covering 166 brands derived from 35 CPG categories, we document several new descriptive empirical generalizations about how price (co)variation changes systematically across planning horizons, categories, and brands.

INFERRING COMPETITIVE RESPONSES

Competitive Inferences at Alternative Planning Cycles: An Overview

Thus far, we have argued that multiple decisions are represented in weekly pricing data and that some of these decisions reflect multiple planning cycles and reaction speeds. In support of this contention, we have shown that pricing interactions differ across planning cycles and that, in general, the dominant pricing interaction does not comport with the sample rate of the data. It is possible to extend these insights by considering the roles of demand and costs in the periodicity of pricing. Accordingly, in this section, we take our analysis a step further and show—again, using spectral decomposition—that the impact of periodicity on statistical inferences about price-setting behavior can be nontrivial. To accomplish this goal, we specify an empirical model to infer demand and competitive response; we then estimate it and offer an explanation of how periodicity affects the results.

Consider demand q_i a function of own prices p_i and competitive prices p_j, $j \neq i$,

$$(3) \qquad q_i = f(p_i, p_j).$$

Under the assumption of constant marginal cost c_i, the first-order condition on profit maximization is that

$$(4) \qquad p_i^* - c_i = -\frac{q_i}{dq_i/dp_i} = -\frac{f(p_i, p_j)}{f_{p_i}(p_i, p_j)}.$$

Substituting a linear demand equation for ease of exposition, where b_{ii} indicates an own-price response coefficient and b_{ij} indicates cross-price response coefficient, we obtain

Table 2

LOGIT REGRESSION RESULTS EXPLAINING THE VARIATION OF PRICES IN THE SHORT, MEDIUM, AND LONG TERM

Variable	Long	t		Medium	t	Short	t	
	Parameter Values by Planning Horizon							
Intercept	−.89	−12.21	**	.00	—	−.65	−8.86	**
Standardized revenue	−.03	−.72		.00	—	.19	5.04	**
Penetration	1.42	2.64	**	.00	—	.47	.88	
Interpurchase time	1.52	2.54	*	.00	—	−.93	−1.55	
Random Effect Variances								
Category/periodicity effects, σ_η^2	.37	8.80	**					
Model error, σ_ξ^2	.44	22.80	**					
Model Fit								
R^2	.58							
Log-likelihood	−251							

*$p < .05$ (two-tailed).
**$p < .01$ (two-tailed).
Notes: Parameters for the medium-term planning horizon are 0, to set a metric.

(5)
$$f_{p_i}(p_i, p_j) = \frac{dq_i}{dp_i} = b_{ii} + b_{ij}\frac{\partial p_j}{\partial p_i},$$

where $b_{ii} < 0$ and $b_{ij} > 0$.

With Bertrand–Nash competition, all players take one another's actions as given; that is, Bertrand–Nash behavior is inferred if $\partial p_j/\partial p_i = 0$. Conversely, if player i believes that raising its price will make player j do the same, then $\partial p_j/\partial p_i > 0$. Given the signs of b_{ii} and b_{ij}, the latter condition is associated with higher margins and therefore is taken as an indication of "soft" competition or "cooperative" behavior. Conversely, finding that $\partial p_j/\partial p_i < 0$ is associated with lower margins and is taken as evidence of noncooperative conduct.

To categorize the nature of competition, one approach has been to simply infer $\partial p_j/\partial p_i$ using the model and empirical data. Such inferences depend on the estimates for b_{ii} and b_{ij}, which, we show, may depend on the periodicity in the data. In contrast, quantities and profit margins do not depend on whether the researcher uses quarterly or weekly data. Therefore, Equations 4 and 5 suggest that the empirical estimate for the sum $b_{ii} + b_{ij} \times (\partial p_j/\partial p_i)$ should be invariant to whether the weekly or quarterly information in the data is used. If b_{ii} is more negative with weekly data than with quarterly data, and if b_{ij} remains largely unaffected, it is easily verified that $(\partial p_j/\partial p_i)$ is higher when it is inferred from weekly rather than quarterly data. The intuition behind this statement is that margins should shrink when price sensitivity increases. Under the preceding assumptions, the equality of margins across regimes of different frequencies forces the inference of cooperation when price response is high. Crucially, this statement builds on the scenario that the only thing that changes across data of different frequencies is own-price response b_{ii}. Although this "all-else-being-equal" condition substantively holds in our empirical example, we do not wish to suggest that it holds in general. Still, if inferences about other parameters change across data of different frequency, it would be a coincidence if the inference about the responses $\partial p_j/\partial p_i$ remained unaffected.

To test this conjecture, we chose a category with two national brands and a private label: shredded cheese. We selected the shredded cheese category because (1) it has a mode in the spectra in both the longer and the shorter terms (see Figure 3), so it is expedient to contrast them, and (2) we have exact knowledge of the variable costs in this category through consultation with a former category manager of one of the major firms in the industry.

To make inferences about competitive response, we adopt a demand specification similar to that of Kadiyali, Chintagunta, and Vilcassim (2000). We specify the demand for brand i as

(6) $\qquad q_i = a_i + b_i p_1 + c_i p_2 + d_i p_3 + g_i \times ddshift_i,$

where p_1 is the price of Kraft shredded cheese, p_2 is the price of Sargento shredded cheese, and p_3 is the price of the store brand. As in the work of Kadiyali, Chintagunta, and Vilcassim (2000, p. 132), we find that the log–log, semilog, and linear demand models are comparable in empirical fit. Our goal is to show that competitive interactions can differ across planning horizons, and a linear model is sufficient for this purpose.

The demand shifters ($ddshift_i$) used here include the total demand of competing brands in the outside stores (i.e., stores other than the one used in estimation) and monthly dummies (Kadiyali, Chintagunta, and Vilcassim 2000). We can show (see Appendix D) that the estimation equations for shredded cheese are given by

(7) $\qquad q_1 = a_1 + b_1 p_1 + c_1 p_2 + d_1 p_3 + g_1 \times ddshift_1,$

$\qquad q_2 = a_1 + b_2 p_1 + c_2 p_2 + d_2 p_3 + g_2 \times ddshift_2,$

$\qquad q_3 = a_3 + b_3 p_1 + c_3 p_2 + d_3 p_3 + g_3 \times ddshift_3,$

$\qquad mp_1 = mc_1 + \gamma_1 q_1,$

$\qquad mp_2 = mc_2 + \gamma_2 q_2,$

$\qquad r_1 = \alpha_1 q_1 + \alpha_2 r_2 + \alpha_3 r_3,$

$\qquad r_2 = \alpha_4 q_2 + \alpha_5 r_2 + \alpha_6 r_3,$ and

$\qquad r_3 = \alpha_7 q_3 + \alpha_8 r_2 + \alpha_9 r_3,$

where mp_i is the manufacturer price of brand $i \in \{1, 2, 3\}$, mc_i is the manufacturer cost, and r_i is the retailer markup for brand i.

In Appendix D, we also show that deviations from the Nash condition in the manufacturer pricing equations (mp_i) are given by $k_i = -(1/\gamma_i) - b_i \neq 0, i \in \{1, 2\}$. Given estimates for the mean and variance of γ_i and b_i, it is possible to obtain estimates for the variance of k_i and therefore test for deviations from Nash. Support for our conjecture that higher elasticities in the short term lead to greater inference of cooperation would manifest as k_i being significantly greater in the short term than in the long term.

To isolate the high- and low-frequency components in the data, we use the spectra to identify the frequency at which the data should be split. The spectral distribution for shredded cheese (see Figure 3) indicates differences in pricing at a frequency less than or equal to 4 weeks and a frequency of more than 4 weeks (with another mode near 14 weeks). The 4-week frequency is consistent with Leeflang and Wittink's (1992) typology for the frequency of long- and short-term pricing interactions. Accordingly, we filtered the shredded cheese category at 4 weeks to select these two frequency components for analysis (see Hamilton 1994).[9] For each frequency component (high and low), we estimated the system of equations in Equation 7 using linear three-stage least squares with the lagged prices and quantities in outside stores and the store under investigation as instruments.

Competitive Inferences at Alternative Planning Cycles: Results

The key parameter estimates from the structural model appear in Table 3, and the own- and cross-price coefficients exhibit face validity. In Table 3, we observe that own-price sensitivity is higher in the high-frequency data than in the low-frequency data. Furthermore, we observe that the cross-price sensitivity does not increase with high-frequency data as much. A potential explanation for this pattern is that the bulk of the short-term price effect comes from shifting demand around in time rather than across brands.

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[9]To explore the robustness of our findings to this 4-week cutoff, we also used decompositions below and above 13 weeks, which yielded the same results.

Table 3

STRUCTURAL MODEL RESULTS

Parameter	Low Frequency		High Frequency	
	Value	t	Value	t
Demand Equations				
Kraft Demand				
Price Kraft (b_1)	−1568.12	−14.00	−2928.24	−27.30
Price Sargento (c_1)	928.11	5.70	1010.44	4.64
Price Dominick's (d_1)	1119.68	6.93	−169.20	−.95
Sargento Demand				
Price Kraft (b_2)	927.19	8.40	536.20	4.11
Price Sargento (c_2)	−1505.25	−10.35	−2005.80	−16.97
Price Dominick's (d_2)	−19.47	−.15	−90.98	−.77
Dominick's Demand				
Price Kraft (b_3)	1496.32	5.40	1706.18	3.75
Price Sargento (c_3)	1012.72	2.71	−485.72	−.84
Price Dominick's (d_3)	−4113.34	−13.13	−4766.57	−10.78
Manufacturer Pricing Rules				
Kraft Rule				
Quantity Kraft (γ_1)	.000582	30.41	.000608	36.09
Sargento Rule				
Quantity Sargento (γ_2)	.000694	32.20	.000777	50.93
Retailer Pricing Rules				
Kraft Rule				
Quantity Kraft (a_1)	−.00017	−15.49	−.00014	−21.29
Markup Sargento (a_2)	.728	10.72	.276	4.60
Markup Dominick's (a_3)	.245	4.73	.031	.69
Sargento Rule				
Quantity Sargento (a_4)	−.00014	−14.94	−.00012	−13.00
Markup Kraft (a_5)	.390	11.23	.170	3.47
Markup Dominick's (a_6) −1.84	.226	6.31	−.079	
Dominick's Rule				
Quantity Dominick's (a_7)	−.00009	−15.39	−.00008	−19.52
Markup Kraft (a_8)	.248	4.88	.069	1.23
Markup Sargento (a_9)	.640	9.20	.003	.04
System-Weighted R^2		.73		.70

To test for deviations from Nash explicitly, we calculate the k_i and its variance using the delta method, or Cramer's theorem (Mela, Gupta, and Lehmann 1997). Table 4 presents these results. Using the low frequencies in the data, we infer that the manufacturers compete in a Nash equilibrium because k_i is not significantly different from 0 for either national brand. Conversely, using the high-frequency data, we infer that the game deviates significantly from Nash because $k_i > 0$ for both national brands. This estimate for k_i implies that cooperation is inferred to be greater in the short run. Thus, this example illustrates that inference about competition depends on which frequencies in the data are used

for analysis. The differences between long and short run are significant, with $t = 9.16$ for k_1 and $t = 3.19$ for k_2.

A tentative explanation for these findings arises for the case in which promotional price response (and elasticity) is greater than regular price response (e.g., Blattberg, Briesch, and Fox 1995). Because our estimates for own-price response for the national brands are 30%–90% greater for high-frequency data, this implies that the inferred level of cooperation should be higher. An alternative explanation is suggested by Lal (1990), who shows that leading brands collude through their promotions to lock out lower-quality brands. Note that implicit collusion in short-term pricing is

Table 4

MANUFACTURING PRICING RULE PARAMETERS

Parameter	Low Frequency		High Frequency	
	Value	t	Value	t
k_1	−150.09	−1.44	1283.50	11.00
k_2	64.33	.39	718.80	5.95

possible when manufacturers learn about promotion schedules in advance from the retailer. In summary, we conclude that our inferences about the nature of the competitive game vary with periodicity and that, in our particular example, more cooperative conduct is inferred with the high-frequency data than with the low-frequency data.

CONCLUSION

Prior literature suggests that price data reflect multiple decisions and multiple decision makers (e.g., retailers and manufacturers) and that these decisions manifest as different pricing interactions across different planning horizons. In this article, we investigated retail prices of CPG products using a decomposition in different planning horizons or price cycles. As such, we follow calls by Kadiyali, Sudhir, and Rao (2001) and Pauwels and colleagues (2004) to analyze pricing interactions across periodicities. In the process, we enumerated five research questions and addressed them as follows.

First, is price variation more common at some frequencies than at others? We find that most of the observed variation in price results not from rapid pricing vacillation but rather from slower, less-frequent pricing movements. This is surprising in light of the emphasis on discounts over regular pricing in both academic and corporate research.

Second, are there dominant frequencies for pricing interactions? If so, do the nature of these interactions differ across frequencies? In our analysis, we find that pricing interactions are more common with regular price changes, but in general, they occur across the entire spectrum. Of additional interest, pricing interactions in the short term sometimes offset those in the long term. For example, prices in the beer category are positively correlated in the long term and negatively correlated in the short term. These interactions cancel out when they are aggregated across frequencies, suggesting the potential for a frequency aggregation bias. Importantly, frequency aggregation differs from time aggregation. Whereas time aggregation issues can be redressed by increasing the sampling frequency of the data, this does not solve the potential for frequency aggregation. Rather, increasing the sample rate adds a high-frequency component to an already existing set of lower-frequency components. To disentangle the price data into meaningful decision cycles, a filtering approach appears promising (see also Deleersnyder et al. 2004; Lamey et al. 2005).

Third, are the frequencies at which prices vary and/or interact related to differences in brand and/or category factors, or are these frequencies common across brands and categories? We find that the price cycles and periodicities at which prices interact have a strong category component that is common to the brands in a category. Our analysis lends some empirical legitimacy to a "category cadence" in pricing decisions. Analyzing the spectra and coherences using descriptors of planning horizons, categories, and brands, we find that coherence is higher for brands from the same manufacturer and those in the same price tier. As the retailer's private-label share in the category decreases, coherence of brands in the category increases. For the spectra, as a brand's share increases and as a category's purchase cycles become shorter, the relative emphasis on short-term pricing compared with regular price changes increases. Our empirical descriptions of the systematic differences in price (co)variation across categories and planning horizons may help anticipate high price (co)variation when it exists.

Fourth, how do price variation and price covariation contrast across frequencies? Do the predominant pricing frequencies and interaction frequencies align? Although pricing variation is greater in the long-term price cycles, pricing interactions (price covariation) are more common across the spectrum. However, a contrast of Figures 3 and 4 suggests that coherence is likely for the most important price cycles. That is, large price cycles are often related across brands.

Fifth, what are some of the potential implications of frequency aggregation for inferences about competitive response? Simply stated, if there are different types of pricing decisions represented at different frequencies, inferences about price responses are potentially confounded. We show how spectral techniques can be used to isolate specific frequencies to infer the nature of competitive response at different planning cycles.

We contend that these findings are novel insofar as there is little research in marketing that explores the role of periodicity in pricing. Given that there are systematic differences in periodicities across brands and categories and that aggregating these frequencies obscures systematic interactions in price, we believe that this study provides an initial step toward disentangling these differences. Above all, we hope that our research is taken as a constructive step in the direction of studying the periodicity of price decision making.

Beyond this domain, we hope that spectral analyses may be fruitfully employed to understand the periodicity of decision making in other marketing contexts. One such application is a recent article by Lemmens, Croux, and Dekimpe (2004), who decompose Granger causality across the spectrum in the context of market expectations, an approach that could also be applied to pricing. In another recent application, Deleersnyder and colleagues (2004) and Lamey and colleagues (2005) link business cycles to sales by using filters to isolate frequencies of interest. Spectral tools can also be applied to other elements of the marketing mix and sales response, such as competitor interactions in advertising or the effect of short-term pricing variation on long-term movement in sales. Another direction would be to develop structural models within the frequency domain to obtain a more complete view of competitive response and to address the issue of dynamics in structural models of competition. Finally, more formal models to integrate periodicity and decision making could yield interesting insights into competitive response. Such insights could be generalized further across categories. We hope that our work sparks more research in this direction.

APPENDIX A: ESTIMATION OF SPECTRAL MODEL

We employ the following procedure to obtain the spectral decomposition of the retail pricing series. The discussion is topical. For more detail, we refer interested readers to the work of Harvey (1975) or, for a more advanced treatment, to the work of Hamilton (1994).

Step 1: Filter the Data

Spectral analysis assumes that the data are stationary (e.g., Nelson and Kang 1981). Augmented Dickey–Fuller tests on the price series in our data indicate that this is often not the case. This will bias the apparent power of the lower

frequencies in the data upward. Therefore, we apply a high-pass filter to the data to eliminate the price cycles with a length equal to or greater than one year. This filter removes both random walk and trend components. The coherence relationships are invariant to filtering when the same linear filter is applied to all the series; however, the spectral density is not (see Fishman 1969; Hassler 1993). Filtering the data controls for linear trends in inflation in costs and prices and annual seasonality (e.g., food costs).

Step 2: Estimate a Vector Autoregressive (VAR) Model

For all prices y_{ict}, for brands $i = 1, ..., K_c$ in a category $c = 1, ..., C$, and for time periods $t = 1, ..., T$, estimate

$$(A1) \qquad \mathbf{y}_{ct}_{(K_c x1)} = \alpha_{c0} + \sum_{s=1}^{L} \Psi_{cs} \times \mathbf{y}_{ct-s} + \mathbf{e}_{ct},$$

where L is the number of lags. The coefficients of this model, ψ_{cs}, are used to compute the spectral decomposition. Given that regular price changes often show little or no variation for up to six months, we allow for long lag shifts of up to 26 weeks (i.e., $L = 26$). We allow for this flexibility because we wish to explore the long-term price interactions for which lower-order VAR models may be less appropriate. Conversely, the higher-order VAR models can be heavily parameterized. There is a $K_c \times K_c$ coefficient matrix ψ to be estimated for each lag. With five brands and 26 periods, this would yield more than 600 parameters for approximately 1900 observations. Therefore, we follow the standard practice of zeroing parameters with t-statistics less than 1.5 (similar to Dekimpe and Hanssens 1999).[10] We proceed in phases by first estimating a VAR model of order 1 and then retaining the parameters with t-statistics greater than 1.5. We then add a second lag and repeat the process. This process continues until all L lags are added to the model.

Step 3: Compute the Spectrum and Cospectrum

The spectrum can be interpreted as the portion of the variance in a price series attributable to a certain frequency. Higher power indicates greater price variation at a given frequency. Then, the cospectrum is analogous to the covariance, and it measures the degree of covariation between two series at a given frequency. We use the coefficient matrices ψ_{cs} to compute the complete power spectrum (Hamilton 1994). Defining $\Omega_e = E(e_{ct}e'_{ct})$, the spectrum at frequency $\omega = 0, ..., \pi$ is a square matrix of size K_c that is equal to

$$(A2) \qquad \mathbf{S}_c(\omega)_{(K_c \times K_c)} = (2\pi)^{-1} \left[\mathbf{I}_{K_c} - \sum_{s=1}^{P} \left\{ \Psi_{cs} e^{-jsw} \right\} \right]^{-1}$$

$$\Omega_e \left[\mathbf{I}_{K_c} - \sum_{s=1}^{P} \left\{ \Psi'_{cs} e^{jsw} \right\} \right]^{-1},$$

where $j = \sqrt{-1}$. In our empirical work, the complex matrix $\mathbf{S}_c(\omega)$ is computed at discrete $\omega = [0, .01 \pi, .02 \pi, ..., \pi]$.

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[10]We also used a cutoff of $t = 1.0$ and $t = .0$ (no cutoff) to assess the sensitivity of our results to the inclusion of more parameters. The results remained essentially identical.

We refer to the resulting series $\{\mathbf{S}_c(\omega)|\omega = 0, ..., \pi\}$ as the "power spectrum." Unlike power spectra estimated from bivariate VAR models, our approach controls for the observed effects of all other brands' prices because we estimate the VAR on the full set of prices.

Step 4: Compute the Coherence

We define for each pair of brands i_c and i'_c the following four quantities:

$$(A3) \qquad s_{ii}^c(\omega) = \mathbf{S}_{ii}^c(\omega),$$

$$s_{i'i'}^c(\omega) = \mathbf{S}_{i'i'}^c(\omega),$$

$$q_{ii'}^c(\omega) = im\{\mathbf{S}_{ii'}^c(\omega)\}, \text{ and}$$

$$c_{ii'}^c(\omega) = re\{\mathbf{S}_{ii'}^c(\omega)\},$$

where $im\{arg\}$ is the imaginary part and $re\{arg\}$ is the real part of its arguments. The factor $q_{ii'}^c$ is called the "quadrature" and $c_{ii'}^c$ is called the "cospectrum." We compute coherence as follows:

$$(A4) \qquad h_{ii'}^c(\omega) \equiv \frac{\left[q_{ii'}^c(\omega)\right]^2 + \left[c_{ii'}^c(\omega)\right]^2}{s_{ii}^c(\omega) s_{i'i'}^c(\omega)}.$$

Note that the coherence measure is symmetric; it is analogous to an R-square measure in regression and measures the strength of association between two series at different frequencies. When we standardize the spectrum $s_{ii}^c(\omega)$ to sum to one across frequencies, it yields the spectral distribution, which we denote as $s_i^c(\omega)$.

Step 5: Compute the Moments of Coherence

Note that there is no confidence interval around Equation A4. To approximate such an interval, we generate 100 draws from the sampling distribution of the estimated VAR parameters of Equation A1, and we use these draws to obtain an empirical distribution for the power spectrum and the resulting measure in Equation A4. This procedure makes it possible to ascertain which frequencies are associated with tightly distributed coherences.

Step 6: Compute the Planning-Cycle-Level Coherence

We aggregate across coherences within a planning cycle by computing a weighted average of the sample mean of coherence at each frequency. The weights we use are the sample variances of the coherence at each frequency. More specifically,

$$(A5) \quad h_{ii'cp} = \sum_{\omega \in p} \frac{var\left[h_{ii'}^c(\omega)\right]}{\sum_{\omega \in p} var\left[h_{ii'}^c(\omega)\right]} \times mean\left[h_{ii'}^c(\omega)\right], \text{ and}$$

$$(A6) \qquad s_{icp} = \sum_{\omega \in p} \frac{var\left[s_i^c(\omega)\right]}{\sum_{\omega \in p} var\left[s_i^c(\omega)\right]} \times mean\left[s_i^c(\omega)\right].$$

APPENDIX B: ESTIMATION OF CROSS-TIME, BRAND, AND CATEGORY EFFECTS

We first specify the coherence regression and discuss its estimation. We then specify the spectral density regression,

which employs the same estimation approach. After collecting the coherences across planning horizons p = (long, medium, short), brand pairs ii′ = 1, ..., N_c, and categories c = 1, ..., C, we then estimate the following regressions for coherence,

(B1) $h_{ii'cp}^h = \delta_{0p}^h + \mathbf{v}_c'\alpha_p^h + \mathbf{z}_{ii'c}'\beta_p^h + \varepsilon_{ii'c}^h + \eta_{cp}^h + \xi_{ii'cp}^h$,

where p is planning horizon, $\mathbf{v}_c'$ are category-level variables, and $\mathbf{z}_{ii'}^c$ are brand-level variables. The η_{cp}^h are within-category/-horizon random effects, the $\xi_{ii'c}^h$ are within-brand-pair effects, and the $\xi_{ii'cp}^h$ are the observational errors. We specify these random effects to account for potential correlations across observations within the same time horizon and category as well as within the same brand pair across time horizons (because, strictly speaking, these coherence estimates are not independent replicates, and failure to accommodate correlations among these repeated measures might overstate the power of the fixed effects). We specify $\xi_{ii'cp}^h \sim$ IIDN $(0, \sigma_\xi^2)$, $\eta_{cp}^h \sim$ IIDN $(0, \sigma_\eta^2)$, and $\varepsilon_{ii'cp}^h \sim$ IIDN $(0, \sigma_{\varepsilon'}^2)$, and we assume that these three errors are independent.

Given that coherence is constrained to lie between 0 and 1, we use the logistic transform of coherence, which is given by $\ln(h_{ii'}(\omega)/[1 - h_{ii'}\{\omega\}])$. Note that the transform is taken before we compute any sample moments. Note also that the $h_{ii'cp}$ are estimates. However, Equation B1 does not consider sample error. To check the robustness of our results to sample error in $h_{ii'cp}$, we replicated the regression using 100 draws from the empirical distribution of $h_{ii'cp}$. The empirical distribution of the estimates for the regression parameters of interest in the second-stage regression differs negligibly from what we reported in the article.

Thus, the component of the covariance structure of the within-category covariation of coherence is the variance structure of $\eta_{cp}^h + \xi_{ii'cp}^h$ across all c; that is,

(B2) $\Lambda_c = \begin{bmatrix} \sigma_\xi^2 + \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \vdots & \ddots & \vdots \\ \sigma_\eta^2 & \cdots & \sigma_\xi^2 + \sigma_\eta^2 \end{bmatrix}_{N_c \times N_c}$, and

$\Lambda = \begin{bmatrix} \Lambda_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \Lambda_C \end{bmatrix}_{N \times N}$,

where $N = \Sigma_{\forall c} N_c$. To obtain the covariance matrix for the system stacked across brand pairs, categories, and planning horizons, we combine Λ with the brand-pair random effects. The resulting covariance matrix is given by

(B3) $\Omega = \mathbf{I}_3 \otimes \Lambda + \sigma_\varepsilon^2 \iota_3 \iota_3' \otimes \mathbf{I}_N$,

where $\mathbf{I}_3$ is a three-dimensional identity matrix, $\mathbf{I}_N$ is an N-dimensional identity matrix, and $\iota_3 \iota_3'$ is a 3×3 matrix of ones. Thus, Ω is dimensioned $3N \times 3N$.

We use maximum likelihood to estimate the model parameters $\theta_h = [\delta_{0p}^h, \alpha_p^h, \beta_p^h, \sigma_\xi^2, \sigma_\eta^2, \sigma_\varepsilon^2]$. We define $e_{ii'cp}^h = h_{ii'cp} - \delta_{0p}^h - \mathbf{v}_c'\alpha_p^h - \mathbf{z}_{ii'c}'\beta_p^h$. We array these residuals across brand pairs and categories so that we obtain an $N \times 1$ vector $\mathbf{e}_p$; let the $N \times 3$ matrix $\mathbf{u} \equiv [\mathbf{e}_0\ \mathbf{e}_1\ \mathbf{e}_2]$. Then, the likelihood function is proportional to

(B4) $L(\theta) \propto |\Omega|^{-.5} \exp\{-.5 \times [\text{vec}(\mathbf{u})' \Omega^{-1} \text{vec}(\mathbf{u})]\}$.

After taking the log of the likelihood and simplifying (Magnus 1982), we can write the log-likelihood function as (ignoring an irrelevant constant),

(B5) $\ln L(\theta) = -.5 \times \ln|\Omega| + .5 \times \text{trace}(\mathbf{u}' \times \Lambda^{-1} \times \mathbf{u})$

            $- .5 \times \text{trace}(\mathbf{u}' \times \mathbf{B} \times \mathbf{u} \times \mathbf{A})$,

where $\mathbf{A} = 1/3\iota_3\iota_3'$, $\mathbf{B} = \Lambda^{-1} - (\Lambda + 3\sigma_\varepsilon^2\mathbf{I}_N)^{-1}$, and the log of the determinant can be shown to equal (using some results on determinants in Magnus [1982] and Searle [1982])

(B6) $\ln|\Omega| = 2\sum_c \left[\ln(\sigma_\xi^2 + N_c\sigma_\eta^2) + (N_c - 1)\ln(\sigma_\xi^2) \right]$

            $+ \sum_c \left[\ln(\sigma_\xi^2 + 3\sigma_\varepsilon^2 + N_c\sigma_\eta^2) + (N_c - 1)\ln(\sigma_\xi^2 + 3\sigma_\varepsilon^2) \right]$.

Note that when the log-likelihood is expressed in this form, it requires only an inversion of a block diagonal $N \times N$ matrix, which is much smaller than Ω. Estimation proceeds by maximizing Equation B5 over θ.

We use a similar procedure to estimate the effect of brand and category factors on the spectral density. Because the spectral density sums to 1 across periodicities, we use the following logit regression equation:

(B7) $s_{icp} = \dfrac{\exp(V_{icp})}{\sum_p \exp(V_{icp})}$, with $V_{icp} = \tilde{\delta}_{0p}^s$

            $+ \mathbf{v}_c'\tilde{\alpha}_p^s + \mathbf{z}_{ic}'\tilde{\beta}_p^s + \tilde{\eta}_{cp}^s + \tilde{\xi}_{icp}^s$.

To normalize, we arbitrarily and without loss in generality subtract from V_{icp} the value V_{ic2} (p = 2 denotes the medium term), so that

(B8) $V_{icp} - V_{ic2} = \delta_{0p}^s + \mathbf{v}_c'\alpha_p^s + \mathbf{z}_{ic}'\beta_p^s + \eta_{cp}^s + \xi_{icp}^s$,

with $\delta_{0p}^s = \tilde{\delta}_{0p}^s - \tilde{\delta}_{02}^s$, $\alpha_p^s = \tilde{\alpha}_p^s - \tilde{\alpha}_2^s$, and so forth. Furthermore, η_{cp}^s and ξ_{icp}^s are normally distributed variables with mean zero and variance σ_η^2 and σ_ξ^2. The parameters of interest can be estimated using the linearization

(B9) $\log\left(\dfrac{s_{icp}}{s_{ic2}}\right) = \delta_{0p}^s + \mathbf{v}_c'\alpha_p^s + \mathbf{z}_{ic}'\beta_p^s + \eta_{cp}^s + \xi_{icp}^s$, p = 1, 3,

which reduces to a linear system analogous to the system for coherence (but with two equations given the sum constraint in the spectral distribution).

APPENDIX C: VARIABLE OPERATIONALIZATION

We operationalize the variables in Table 1 as follows: "Same manufacturer" is an indicator variable that assumes the value of 1 if the two brands in a given category are produced by the same manufacturer and 0 if otherwise. Note that the same manufacturer is not equivalent to the same brand. "Private label" is an indicator variable that assumes the value of 1 if one of the brands in the pair is a private-label brand. "Revenue differential" is the absolute difference between the mean sales (across time) for the brand

pairs, and "price differential" reflects the mean per unit absolute price difference over time. We based both sales differential and price differential on standardized series to make the variables comparable across categories (as units of volume differ across categories). We substitute "standardized revenue" within a category for revenue deferential in the spectral density regression.

We measure "concentration" using the Hirschman–Herfindahl indexes. We define "volatility" as the sum of SKU births and deaths in the data, expressed as a fraction of the number of SKUs. We determined a birth by the appearance of a SKU sometime over the duration of the data. We determined a death by the disappearance of a SKU before the end of the data. We convert each of these to a percentage by dividing by the total number of SKUs. "Private-label share" represents the share of the store brand computed over the duration of the data. Finally, we obtain "interpurchase time" (in days) and "penetration" (as the percentage of consumers using the category) from the Information Resources Inc. factbook.

APPENDIX D: SPECIFICATION AND ESTIMATION OF STRUCTURAL MODEL

We use the following demand equations for three brands (ignoring demand shifters for the sake of explication),

$$(D1) \qquad q_i = a_i + b_ip_1 + c_ip_2 + d_ip_3.$$

Then, profits for Brand 1 are given by $(mp_1 - mc_1)q$, where mp_1 is the manufacturer price of Brand 1 and mc_1 is the manufacturer cost. The derivative with respect to p_1 is given by $(mp_1 - mc_1)'q_1 + (mp_1 - mc_1)q'_1$. This implies $(mp_1 - mc_1)(b_1[\partial p_1/\partial mp_1] + c_1[\partial p_2/\partial mp_1] + d_1[\partial p_3/\partial mp_1]) + q_1 = 0$. If $p_1 = mp_1 + r_1$, where p is retail price and r is markup, then $(mp_1 - mc_1)(b_1[1 + \partial r_1/\partial mp_1] + c_1[\partial p_2/\partial mp_1] + d_1[\partial p_3/\partial mp_1]) + q_1 = 0$. Setting $\partial r_1/\partial mp_1 = t_1$, $\partial p_2/\partial mp_1 = t_2$, and $\partial p_3/\partial mp_1 = t_3$ implies that $mp_1 = mc_1 - q_1(b_1 + b_1t_1 + c_1t_2 + d_1t_3)^{-1}$. When t_1, t_2, and t_3 are 0, we observe Nash; however, these parameters are not separately identified. Setting $k_1 = b_1t_1 + c_1t_2 + d_1t_3$, we obtain $mp_1 = mc_1 - q_1(b_1 + k_1)^{-1}$. Thus, if k_1 differs from 0, the game is not Nash (Kadiyali, Chintagunta, and Vilcassim 2000). Note that $k_1 = 0$ is a necessary but not sufficient condition for Nash. Setting $k_1 = -(1/\gamma_1) - b_1$ yields

$$(D2) \qquad mp_1 = mc_1 + \gamma_1q_1,$$

which is the estimation equation. After estimating γ_1 and b_1, the variance of k_1 can be inferred using the delta method. A similar equation holds for the other brands. Note that γ is a measure of relative margins insofar as the manufacturer margin $mp_1 - mc_1 = \gamma_1q_1$.

Retailer profits are given by $r_1q_1 + r_2q_2 + r_3q_3$. Selecting r_1 to maximize profits implies that $r'_1q_1 + r_1q'_1 + r'_2q_2 + r_2q'_2 + r'_3q_3 + r_3q'_3 = 0$. Thus, $q_1 + r_1(b_1[\partial mp_1/\partial r_1 + 1] + c_1\partial mp_2/\partial r_1 + d_1\partial mp_3/\partial r_1) + r_2(b_2[\partial mp_1/\partial r_1 + 1] + c_2\partial mp_2/\partial r_1 + d_2\partial mp_3/\partial r_1) + r_3(b_3[\partial mp_1/\partial r_1 + 1] + c_3\partial mp_2/\partial r_1 + d_3\partial mp_3/\partial r_1) = 0$. Again, noting that we cannot separately identify the conduct parameters, we obtain $r_1 = -(b_1 + k_4)^{-1}q_1 - (b_2 + k_5)(b_1 + k_4)^{-1}r_2 - (b_3 + k_6)(b_1 + k_4)^{-1}r_3$, where $k_4 = b_1\partial mp_1/\partial r_1 + c_1\partial mp_2/\partial r_1 + d_1\partial mp_3/\partial r_1$, $k_5 = b_2\partial mp_1/\partial r_1 + c_2\partial mp_2/\partial r_1 + d_2\partial mp_3/\partial r_1$, and $k_6 = b_3\partial mp_1/\partial r_1 + c_3\partial mp_2/\partial r_1 + d_3\partial mp_3/\partial r_1$. When k_4, k_5, $k_6 = 0$, this is

Nash (Kadiyali, Chintagunta, and Vilcassim 2000). Setting $k_4 = (-1 - \alpha_1b_1)/\alpha_1$, $k_5 = -b_2 - \alpha_2(b_1 + k_4) = -b_2 + \alpha_2/\alpha_1$, and $k_6 = -b_3 - \alpha_3(b_1 + k_4) = -b_3 - \alpha_3/\alpha_1$ yields the estimation equation,

$$(D3) \qquad r_1 = \alpha_1q_1 + \alpha_2r_2 + \alpha_3r_3,$$

which is linear. Together, equations D1–D3 form the system in Equation 7.

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